AD P000339

A ONE-DIMENSIONAL PROCEDURE FOR ESTIMATING THE PERFORMANCE OF EJECTOR NOZZLES

by

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The exhaust nozzles of modern turbine engines are often configured as ejectors. The turbine exit gas stream functions as the primary or driving flow, while the secondary flow may be ambient or ram air, fan discharge, or accumulated bleed and leakage flows. The procedure described in this paper was developed to provide a quick performance estimate for ejector nozzles at typical in-flight operating conditions.

The paper begins with a review of compound compressible flow theory, then shows how the choked mass flow and thereby, the ejector pumping capacity are found. Analysis of unchoked operation is described and thrust equations for choked and unchoked operation are derived.

The final section of the paper deals with the prediction of mixed flow performance for both constant pressure and constant area mixing.

Manuscript Received August 1979. The work was performed under AFAPL Work Unit No. 30660602.

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I. INTRODUCTION

The exhaust nozzles of modern aircraft are often required to flow more than one gas stream. Installations flowing two and three gas streams are common. The extra streams may be nozzle cooling air, a relatively small fraction of the primary flow, and/or fan discharge or ram air for thrust augmentation, which may be several times the primary flow. Proper design and accurate performance assessment of such a nozzle requires an understanding of the behavior of multiple flows. Reference 1 develops the basic theory for compound compressible flow, which will be outlined briefly in the succeeding paragraphs. To keep things simple, the discussion will consider only the primary stream and one secondary stream. Analysis of three or more streams obviously would follow the same general procedure.

For nozzle pressure ratio sufficiently high that the primary stream is supersonic, a method of characteristics solution of the primary stream coupled to a one dimensional treatment of the secondary, as outlined in Reference 2, yields results which agree well with measurements. For lower pressure ratios where the primary stream is subsonic or barely supersonic, the method of characteristics will not work and an alternative method of solution is needed. The one dimensional analysis presented herein was developed to fill this gap and serve as a supplement to the more accurate method of characteristics procedure.

II. DISCUSSION

Four concepts are fundamental to the operation of ejector nozzles - compound choking, the pumping characteristic, operation in the unchoked mode, and resultant thrust. These are presented in turn.

Consider the ejector shown in Figure 1, which consists of a primary stream at a specified total temperature and pressure and a secondary stream at some other pressure and temperature, flowing through the ejector shroud into the ambient air, whose pressure is known. Assume that the ambient pressure is just slightly lower than the primary and secondary total pressures so that both streams flow slowly through the nozzle and out the exit. The ejector at this condition is unchoked, exactly as would a single flow nozzle be. Now picture the situation as we lower the ambient pressure while holding the other parameters constant. The velocity and the mass flow of the two streams increase, just as for a single flow nozzle, until we reach a value of back pressure below which the mass flow through the nozzle does not increase. Again analogous to the single flow case, we define this to be the point of compound choking. Were there but a single stream, the Mach number at the minimum flow area would be unity. But can we expect two streams to both be sonic at the minimum area? As will be shown below, the usual case is that the primary stream is supersonic while the secondary is subsonic, yet the nozzle is choked; decreasing the back pressure yields no increase in mass flow.

The derivation of the governing one dimensional flow equations assumes (1) the two streams do not mix, and (2) the static pressures of the two streams are equal at any axial station. For any one dimensional gas flow,

$$\frac{dP}{P} = \frac{k\,M^2}{1 - M^2}\,\frac{dA}{A} \tag{1}$$

or

$$\frac{1}{P}\,\frac{dP}{dx} = \frac{k\,M^2}{1 - M^2}\,\frac{1}{A}\,\frac{dA}{dx} \tag{2}$$

solving for $\frac{dA}{dx}$

$$\frac{dA}{dx} = \frac{A}{k}\left(\frac{1 - M^2}{M^2}\right)\frac{1}{P}\,\frac{dP}{dx} \tag{3}$$

For the two streams with $A = Ap + As$ and $Pp = Ps = P$,

$$\frac{dA}{dx} = \frac{dAp}{dx} + \frac{dAs}{dx} \tag{4}$$

$$= \left[\frac{Ap}{kp}\left(\frac{1}{Mp^2} - 1\right) + \frac{As}{ks}\left(\frac{1}{Ms^2} - 1\right)\right]\frac{1}{P}\,\frac{dP}{dx} \tag{5}$$

Define the compound flow indicator β:

$$\beta = \frac{Ap}{kp}\left(\frac{1}{Mp^2} - 1\right) + \frac{As}{ks}\left(\frac{1}{Ms^2} - 1\right) \tag{6}$$

In one dimensional flow, choking occurs when $\frac{dA}{dx} = 0$ and $\frac{dP}{dx} \neq 0$ (In a choked nozzle, $\frac{dP}{dx}$ is negative at the throat.); thus for two streams to be choked, β must be zero. Positive β indicates unchoked flow, sensitive to the exit pressure, while negative β indicates the opposite. Observe also that unchoked flow with a supersonic stream could exist as well as the aforementioned choked flow with a subsonic stream.

Thus far we have defined a variable, β, which is zero when $\frac{dA}{dx}$ is zero and $\frac{dP}{dx}$ is not. By analogy with the single flow case we have stated but not proven that a multiple stream flow is choked when β is zero. Before we can prove this, we need to determine which pair of Mach numbers Mp and Ms will cause β to be zero. This in turn will lead to the definition of the pumping characteristic, Pos/Pop.

Referring again to Figure 1, assume that Amin/Aop, Aex/Aop, Pamb/Pos and

W/To)s/W/To)p are known. We want to find the values of Mp and Ms and thereby the ratio Pos/Pop which will cause β to be zero. For each stream we can write

$$\frac{W}{A} = \sqrt{\frac{kg}{RTo}}\ PM\sqrt{1 + \frac{k - 2}{2} M^2} \tag{7}$$

For two streams with equal static pressures

$$\frac{W\sqrt{To})s}{W\sqrt{To})p} = \frac{As}{Ap}\frac{Ms}{Mp}\sqrt{\frac{Rp}{Rs}\frac{ks}{kp}}\ \frac{\sqrt{1 + \frac{ks - 1}{2} Ms^2}}{\sqrt{1 + \frac{kp - 1}{2} Mp^2}} \tag{8}$$

Also for each stream we can write

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{2}{k + 1}\left(1 + \frac{k - 1}{2} M^2\right)\right]^{\frac{k + 1}{2(k - 1)}} \tag{9}$$

At the minimum area

$$\frac{Amin}{Ap*} = \frac{Ap}{Ap*} + \frac{As}{Ap*} \tag{10}$$

If the primary stream is choked at Aop, Ap* = Aop.

Then

$$\frac{As}{Ap*} = \frac{Amin}{Aop} - \frac{Ap}{Ap*} \tag{11}$$

and

$$\frac{As}{Ap} = \frac{As}{Ap*}\frac{Ap*}{Ap} \tag{12}$$

thus

$$\frac{As}{Ap} = \frac{Amin}{Aop}\frac{Ap*}{Ap} - 1 \tag{13}$$

We now have a system of two equations, (6) and (8), in two unknowns, Mp and Ms, which may be solved with the aid of equations (9) and (13). Once Ms and Mp are found, the ratio Pos/Pop is calculated from

$$\frac{Pos}{Pop} = \frac{\left(1 + \frac{ks - 1}{2} Ms^2\right)^{\frac{ks}{ks - 1}}}{\left(1 + \frac{kp - 1}{2} Mp^2\right)^{\frac{kp}{kp - 1}}} \tag{14}$$

Ms and Mp are found iteratively, by the method of false position as follows:

a. Guess two values of Mp, one just supersonic, the other around 2 or 2.5.

b. From equation (13), each Mp will yield an As/Ap.

c. These, plugged into equation (8) will give a pair of Ms.

d. Equation (6) then will yield two β, positive value corresponding to the lower Mp, and a lower value corresponding to the higher Mp. If the second β is negative, the desired value of β (zero) is bracketed and the solution can proceed as described in any text on numerical analysis. If not, repeat with a higher Mp till a negative β results, then proceed.

e. Once the Ms and Mp which make β zero are found, the pumping characteristic Pos/Pop is computed from equation (14).

III. UNCHOKED OPERATION

Unchoked operation implies that the compound flow system is sensitive to the back (ambient) pressure. The underlying assumptions are that at any axial station the static pressures of the two streams are equal to each other and at the exit plane, they are equal to the ambient pressure. Knowing the corrected flow ratio $Ws\sqrt{Tos}/Wp\sqrt{Top}$ and the overall pressure ratio Pop/Pamb, one calculates in turn

a. The exit area occupied by the primary stream,

b. The exit area available to the secondary stream,

c. The total pressure which will force Ws through the available exit area. As Pop/Pamb decreases, there comes a point where Pos falls below ambient and ejector action ceases.

These three steps are accomplished as follows, remembering that the primary stream is assumed to be always sonic at Aop:

a. Select the primary pressure ratio, Pop/Pamb. This will always be lower than the pressure ratio at which the ejector compound chokes. From the pressure ratio, calculate the primary exit Mach number, Mp, and area ratio, Ap/Ap* (=Ap/Aop), from the isentropic relations.

b. The area available to the secondary stream is found from

$$\frac{As}{Ap^*} = \frac{Ae}{Ap^*} - \frac{Ap}{Ap^*} \tag{15}$$

and equation (12).

c. Combining equations (8) and (14) yields

$$\frac{Ws}{Wp}\sqrt{\frac{Tos}{Top}} = \frac{As}{Ap}\,\frac{Pos}{Pop}\,\frac{\sqrt{\frac{ks}{Rs}}}{\sqrt{\frac{kp}{Rp}}}\,\frac{Ms}{Mp}\,\frac{\left(1 + \frac{ks - 1}{2}\,Ms^2\right)^{-\frac{ks + 1}{2(ks - 1)}}}{\left(1 + \frac{kp - 1}{2}\,Mp^2\right)^{-\frac{kp + 1}{2(kp - 1}}} \tag{16}$$

The only unknown is Pos/Pop, which is found iteratively. Guessing a value for Pos/Pop also gives a value for Pos/Pamb since Pop/Pamb is specified in step a. From this, Ms is Calculated and finally equation (16) yields an estimate of the corrected flow ratio. This is compared with the given value and Pos is adjusted as necessary until the estimate agrees with the specification to an acceptable degree.

IV. THRUST CALCULATION

The basic thrust equation is the nondimensional form for gross thrust,

$$\frac{F}{PoA*} = k\sqrt{\frac{2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\sqrt{1-\left(\frac{Pe}{Po}\right)^{\frac{k-1}{k}}} + \frac{Ae}{A*}\left(\frac{Pe}{Po}-\frac{Pamb}{Po}\right) \qquad (17)$$

Equation (17) is written for each stream and the thrust of the secondary is adjusted to refer to PoA* of the primary, via

$$\text{Adjust} = \frac{Pos}{Pop}\,\frac{As*}{Ap*}$$

$$= \frac{Pos}{Pop}\,\frac{Ap}{Ap*}\,\frac{As}{Ap}\,\frac{As*}{As} = \frac{Ws\sqrt{Tos}}{Wp\sqrt{Top}} \qquad (18)$$

$$\frac{Ftotal}{PopA_p^*} = \frac{Fp}{PopA_s^*} + \frac{Fs}{PosA_s^*} \times \text{Adjust} \qquad (19)$$

Two thrust coefficients are calculated, the first being total thrust divided by the ideal total thrust, and the second being the total thrust divided by the ideal thrust of the primary, viz

$$Cv = \frac{Ftotal}{Ftotal\ ideal} \qquad (20)$$

$$Cvp = \frac{Ftotal}{Fp\ ideal} \qquad (21)$$

Where ideal thrust is obtained by setting Pe = Pamb in equation (17).

For unchoked flow, the thrust is calculated from

$$\frac{F}{PoA*} = \frac{P}{Po}\,\frac{A}{A*}\,kM^2 \qquad (22)$$

Which is equivalent to (17) with Pe = Pamb. Cv, Cvp, and Adjust are the same as above.

V. MIXING ANALYSIS

The foregoing discussions have assumed that the two streams pass through the ejector shroud without mingling, i.e. as if they were separated by a flexible membrane. It is possible to estimate in a similar manner the effect of complete mixing of the two streams.

5.1 Assumptions

We know the ejector geometry as before and we are given the bypass ratio B = Ws/Wp and the ratio of stagnation temperatures, Tos/Top. From these, we can form the corrected flow ratio as WR = B Tos/Top. Finally, assume that the ratio of specific heats, k, is constant and the same for both streams and that the static pressures for both streams are equal.

5.2 Analysis

As before, we shall render all quantities dimensionless by referencing them to the appropriate primary stream variable. Then, by applying the conservation laws for mass, momentum, and energy, we can develop a description of the mixed stream.

Energy: Summing the energies of the two streams yields

$$Tom = \frac{WpTop + WsTos}{Wp + Ws} \qquad (23)$$

Or, in terms of WR and B

$$\frac{Tom}{Top} = \frac{B + (WR)^2}{B(1 + B)} \qquad (24)$$

Momentum:

$$(1 + kM^2m)\,\frac{PmAm}{PopAp*} = \frac{Ps}{Pop}\,\frac{As}{Ap*}\,(1 + kM^2x) + \frac{Ap}{Ap*}\,(1 + kM^2p) \qquad (25)$$

$$= \frac{F}{PopAp*} \qquad (25a)$$

Mass: Writing equation (7) for the mixed stream yields

$$\frac{Wp(1 + B)}{PmAm} \sqrt{Tom} = \sqrt{\frac{kg}{R}}\ Mm \sqrt{1 + \frac{k - 1}{2} Mm^2}$$

In dimensionless form

$$\frac{Wp\sqrt{Top}}{PopAp^*} \frac{PopAp^*}{PmAm} (1 + B)\sqrt{\frac{Tom}{Top}} = \sqrt{\frac{kg}{R}}\ Mm \sqrt{1 + \frac{k - 1}{2} Mm^2} \tag{26}$$

Recall Fliegner's formula

$$\frac{W}{A^*} \frac{\sqrt{To}}{Po} = \sqrt{\frac{kg}{R}} \sqrt{\left(\frac{2}{k + 1}\right)^{\frac{k + 1}{k - 1}}} \tag{27}$$

Substituting equations (24) and (27) into (26) yields

$$\sqrt{\left(\frac{2}{k + 1}\right)^{\frac{k + 1}{k - 1}}}\ \frac{PopAp^*}{PmAm} (1 + B) \sqrt{\frac{B + (WR)^2}{B(1 + B)}} = Mm \sqrt{1 + \frac{k - 1}{2} Mm^2} \tag{28}$$

From equation (25a) we have

$$\frac{PopAp^*}{PmAm} = \frac{PopAp^*}{F} (1 + kMm^2)$$

Substituting this into equation (28),

$$\frac{PopAp^*}{F} \sqrt{\left(\frac{2}{k + 1}\right)^{\frac{k + 1}{k - 1}}\ \frac{(1 + B)(B + (WR)^2)}{B}}\ (1 + kMm^2) = Mm \sqrt{1 + \frac{k - 1}{2} Mm^2} \tag{29}$$

Define a temporary variable Q

$$Q = \left(\frac{PopAp^*}{F}\right)^2 \left(\frac{2}{k + 1}\right)^{\frac{k + 1}{k - 1}} \frac{(1 + B)(B + (WR)^2)}{B} \tag{30}$$

Then

$$\sqrt{Q}\ (1 + kMm^2) = Mm \sqrt{1 + \frac{k - 1}{2} Mm^2} \tag{31}$$

If we square equation (31), we obtain a quadratic in Mm^2, from which

$$Mm^2 = \frac{1 - 2kQ + \sqrt{(2kQ - 1)^2 - 4(k^2Q - \frac{k - 1}{2})Q}}{2(k^2Q - \frac{k - 1}{2})} \tag{32}$$

The negative root yields the subsonic solution and the positive root the supersonic.

Finally, we have the choice of constant area or constant (static) pressure mixing. In either case, we use equation (25a). In the first case Am is given by the geometry, usually Ap + As. Then

$$\frac{Pm}{Pop} = \left(\frac{Pm}{Pop}\cdot\frac{Am}{Ap*}\right)\left(\frac{Ap*}{Am}\right) = \left(\frac{F}{PopAp*}\right)\frac{Ap*/Am}{(1 + kMm^2)} \qquad (33)$$

In the second case, Pm = Ps and we compute Am from

$$\frac{Am}{Ap*} = \left(\frac{PmAm}{PopAp*}\right)\left(\frac{Pop}{Pm}\right) = \left(\frac{F}{PopAp*}\right)\frac{(Pop/Ps)}{(1 + kMm^2)} \qquad (34)$$

The thrust in either case is calculated from equation 17 and referenced to PopAp* via

$$\frac{Fm}{PopAp*} = \left(\frac{Fm}{PomAm*}\right)\left(\frac{Pom}{Pop}\right)\left(\frac{Am}{Ap*}\right)\left(\frac{Am*}{Ae}\right)\left(\frac{Ae}{Am}\right) \qquad (35)$$

and we can define a thrust coefficient

$$Cfm = \frac{\dfrac{Fm}{PopAp*}}{\dfrac{Fp}{PopAp*} + \dfrac{Fs}{PopA_p^*}} \qquad (36)$$

where the denominator is calculated as described previously.

VI. RESULTS & CONCLUSIONS

The one-dimensional analysis described above has been implemented in a Fortran computer program called EFFORT. For completeness, EFFORT will also compute the performance of nozzles with no secondary flow. Copies of the source deck may be obtained from the author.

EFFORT runs very quickly (more than 20 cases per second) and the results of the compound choking analysis agree very well with the method of characteristics solution described in reference 2. Therefore one can conduct parametric studies with EFFORT, then use a more detailed analysis when the ejector configuration is better defined. EFFORT, being a one-dimensional, inviscid analysis, will yield an upper bound on expected performance.

The unchoked analysis in EFFORT extends the performance caluclation to lower pressure pressure ratios than an MOC analysis can handle, but which may still be of interest.

The mixing calculations are also estimates. They were included for completeness so that one could compare the mixed performance with the compound choked performance. In neither instance is the mechanism considered, only what the final result would be.

Constant area mixing yields thrust equal to or slightly poorer than the unmixed thrust. The constant pressure mixing results can be misleading because the area required to maintain the mixing pressure is usually smaller than the specified Amin. Therefore, the constant pressure analysis is solving a different problem. One can iterate to a solution by adjusting WR and ARmin until Amix agrees with Amin, but that iteration is not built into EFFORT.

Note that EFFORT deals with gross thrust only. Equation (20) compares

the actual gross thrust of the ejector to the thrust available from the two streams expanded ideally and therefore provides the most meaningful measure of performance. Ejector performance is sometimes quoted in terms of an "augmentation ratio," the definition of which varies from investigator to investigator. The thrust coefficient defined by equation (21) can be considered an augmentation ratio in that it compares the actual ejector thrust to the ideal thrust of the primary stream. One could also define an augmentation ratio which compares the actual ejector thrust to the thrust of the primary stream expanded to the ejector exit area. This ratio would be larger than from equation (21) except wehre the exit area is appropriate to the pressure ratio. Regardless of the measure of performance one adopts, it is important to understand that the thrust increase from the secondary mass flow is much smaller than the increase from relieving the overexpansion of the primary stream. By filling the excess area of the shroud, the secondary stream allows the primary to operate at an area ratio more suitable to the pressure ratio and reduces the PA in terms in the thrust equation.

Finally, one must clearly understand that no ejector can increase the net thrust of a propulsion system except by relieving overexpansion. Thrust is momentum. Gross thrust is the momentum of the exhaust stream(s) and net thrust is gross thrust less their incoming momentum. Consider the control volume in Figure 2. The two streams enter at the left. The primary stream gains momentum because the engine adds work to it, drawing energy from the fuel. The secondary stream, if it gains momentum at all, has to gain it from the primary stream, for there is no other source. Therefore, regardless of the amount of secondary flow, the net momentum exiting the right side of the control volume is constant.

REFERENCES

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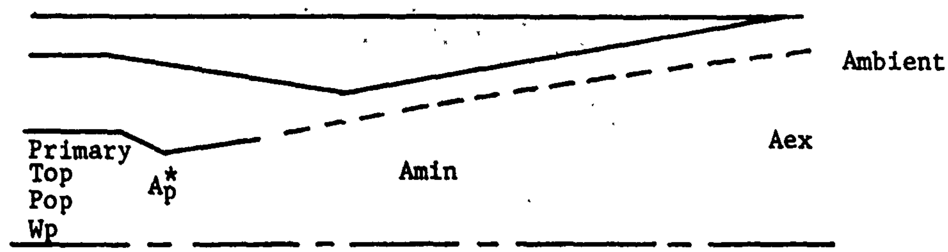

Figure 1

Ejector Nomenclature

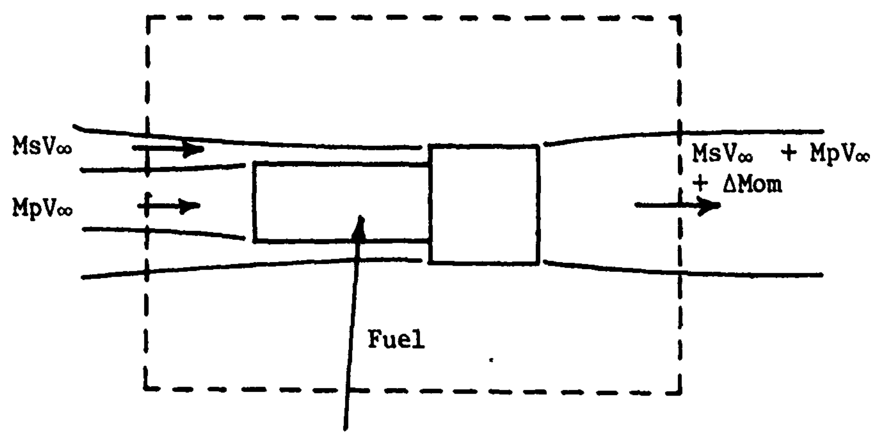

Figure 2

Control Volume